$$\frac{\omega\rho'}{1+\sqrt{\rho'}}+\omega=\frac{S}{t}=\frac{\omega t'}{t}. \quad \therefore\ \rho'-\frac{t'-t}{t}\sqrt{\rho'}=\frac{t'-t}{t};$$

$$\therefore\ \sqrt{\rho'}=\frac{1}{2t}\left\{(t'-t)+\sqrt{t'^2-3t^2+2tt'}\right\}.$$

$$\therefore\ \rho'=\frac{t'-t}{2t^2}\left\{(t'+t)+\sqrt{t'^2-3t^2+2tt'}\right\} \quad \text{..........................(2).}$$

Suppose Venus transits the sun's disc at such a rate that the sun's apparent diameter would be traversed in $7\frac{1}{3}$ hours, and at the same time the sun in his annual course moves through a distance equal to his own apparent diameter in 12 hours. Required (1) the distance from Venus to the sun, the earth's distance being unity, and (2) the distance from the earth to the sun, Venus's distance being unity.

Now $t=7\frac{1}{3}$, $t'=12$; hence for first case substituting in (1)

$$\rho=\tfrac{1}{2}\left(\tfrac{29}{7}-\sqrt{\tfrac{51}{7}}\right)=.721824.$$

For the second case substitute in (2)

$$\rho'=\tfrac{7}{484}\{58+\sqrt{1428}\}=1.38538.$$

(The above is suggested in Proctor's Geometry of the Cycloid.)

A PROPOSITION IN DETERMINANTS.

By ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi.

THEOREM.—The product of two numbers, each the sum of four squares, is the sum of eight squares.

$$\begin{vmatrix} a+b\sqrt{-1} & -c+d\sqrt{-1} \\ c+d\sqrt{-1} & a-b\sqrt{-1} \end{vmatrix} \times \begin{vmatrix} \alpha+\beta\sqrt{-1} & -\gamma+\delta\sqrt{-1} \\ \gamma+\delta\sqrt{-1} & \alpha-\beta\sqrt{-1} \end{vmatrix}$$

$$=\begin{vmatrix} a+b\sqrt{-1} & -c+d\sqrt{-1} & 0 \\ c+d\sqrt{-1} & a-b\sqrt{-1} & 0 \\ 0 & 0 & 1 \end{vmatrix} \times(-1) \begin{vmatrix} \alpha+\beta\sqrt{-1} & 0 & -\gamma+\delta\sqrt{-1} \\ \gamma+\delta\sqrt{-1} & 0 & \alpha-\beta\sqrt{-1} \\ 0 & 1 & 0 \end{vmatrix}$$

$$=(-1)\begin{vmatrix} a\alpha-b\beta+(a\beta+b\alpha)\sqrt{-1} & a\gamma-b\delta+(a\delta+b\gamma)\sqrt{-1} & -c+d\sqrt{-1} \\ c\alpha-d\beta+(c\beta+d\alpha)\sqrt{-1} & c\gamma-d\delta+(c\delta+d\gamma)\sqrt{-1} & a-b\sqrt{-1} \\ -\gamma+\delta\sqrt{-1} & \alpha-\beta\sqrt{-1} & 0 \end{vmatrix}$$

$$= \begin{vmatrix} c\alpha-d\beta+(c\beta+d\alpha)\sqrt{-1} & c\gamma-d\delta+(c\delta+d\gamma)\sqrt{-1} \\ -c\gamma+d\delta+(c\delta+d\gamma)\sqrt{-1} & c\alpha-d\beta-(c\beta+d\alpha)\sqrt{-1} \end{vmatrix}$$

$$+ \begin{vmatrix} a\alpha-b\beta+(a\beta+b\alpha)\sqrt{-1} & a\gamma-b\delta+(a\delta+b\gamma)\sqrt{-1} \\ -a\gamma+b\delta+(a\delta+b\gamma)\sqrt{-1} & a\alpha-b\beta-(a\beta+b\alpha)\sqrt{-1} \end{vmatrix}$$

or $(a^2+b^2+c^2+d^2)(\alpha^2+\beta^2+\gamma^2+\delta^2)=(c\alpha-d\beta)^2+(c\beta+d\alpha)^2$
$+(c\gamma-d\delta)^2+(c\delta+d\gamma)^2+(a\alpha-b\beta)^2+(a\beta+b\alpha)^2+(a\gamma-b\delta)^2+(a\delta+b\gamma)^2$.

Euler's Theorem is an easy corollary of this, and *vice-versa*.

University of Mississippi, March, 1896.

A METHOD OF SOLVING QUADRATIC EQUATIONS.

By Prof. HENRY HEATON, M. Sc., Atlantic, Iowa.

Let it be required to solve the equation

$$ax^2+bx+c=0 \qquad (1).$$

Transposing the middle term we have

$$ax^2+c=-bx \qquad (2).$$

Squaring, $a^2x^4+2acx^2+c^2=b^2x^2$ ....(3).

Subtracting $4acx^2$, $a^2x^4-2acx^2=(b^2-4ac)x^2$ ....(4).

Extracting the square root, $ax^2-c=\pm(\sqrt{b^2-4ac})x$ ....(5).

Adding equation (2), $2a^2x^2=(-b\pm\sqrt{b^2-4ac})x$ ....(6).

Whence $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Let it be required to solve the equation $3x^2-2x=21$.

Transposing $2x$ to the second member and 21 to the first, the equation becomes

$$3x^2-21=2x \qquad (7).$$